

The Galaxy Ending Supernova of Star Trek

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1 Introduction

Let me preface this document by saying that I thoroughly enjoyed the recent Star Trek movie. There were, however, certain aspects of the movie that were exceptionally unreasonable. I decided to work through the calculations to get an idea of what things would be really like for a super massive star exploding in our galaxy as compared to the implication that a supernova could destroy the earth as suggested by the movie. Here are some approximate values I will be using:

$$\begin{aligned}M_{\odot} &= 1.989 \times 10^{30}\text{kg} \\R_{\odot} &= 6.955 \times 10^8\text{m} \\L_{\odot} &= 3.9 \times 10^{26}\text{W} \\M_{\oplus} &= 5.974 \times 10^{24}\text{kg} \\R_{\oplus} &= 6.378 \times 10^6\text{m} \\m_{\star} &= 500M_{\odot} \\distance_{\star} &= 4 \text{ ly}\end{aligned}$$

The model I am using is a star exploding near our own solar system. Some of the values I am using are a little unreasonable, but they extend the physics to the most extreme that I deem justifiable in the current context (actually a little beyond). For instance, I did a quick search on wikipedia, and it suggested the maximum known star mass to be $200M_{\odot}$. I chose to use $500M_{\odot}$ to give the writers a little room to breath, as it were. I also will assume a complete mass to energy transformation, which is completely unrealistic, and I will assume that the energy will not be absorbed or redirected in the interstellar medium. The proximity of the star was chosen to represent the star nearest our own planet to give us something to work with. One final assumption worthy of note is that we are assuming the energy is ejected with a perfect spherical symmetry, but we know that in some scenarios this is not true (for instance if we were referring to Gamma Ray Bursts). Since, however, the exploding star was referred to and depicted as a supernova with said spherical symmetry, I feel that such an assumption is justifiable.

2 Star Energies

According to relativity, we know that $E = mc^2$. Again, we are assuming a complete matter to energy conversion. We know that this is very unrealistic. I imagine it would be much closer to a 1 percent conversion. This however is based on a very loose guess. Again this value gives the writers the benefit of the doubt. Given this assumption, the energy released by this star would be:

$$\begin{aligned} E_{\star} &= m_{\star} \times c^2 \\ &= 500M_{\odot} \times c^2 \\ &\simeq 8.938 \times 10^{49} \text{J} \end{aligned}$$

Now, since there is some distance separating the exploding star from our sun and planet, most of the energy will not have a direct impact on us. Only a fraction of the energy released will actually impact the given target. The amount of energy that will be imparted is equivalent to the total energy times the area of the target divided by the surface area of the energy at the target. We'll call this the effective energy.

$$E_{eff} = E_{\star} \times \frac{A_{target}}{A_{surf}}$$

For the Sun:

$$\begin{aligned} E_{\odot} &= E_{\star} \times \frac{A_{\odot}}{A_{surf}} \\ &= E_{\star} \times \frac{\pi \times R_{\odot}^2}{4 \times \pi \times (4 \text{ ly})^2} \\ &= 8.938 \times 10^{49} \times \frac{\pi \times (6.955 \times 10^8 \text{ m})^2}{4 \times \pi \times (4 \text{ ly})^2} \\ &\simeq 7.5 \times 10^{33} \text{J} \end{aligned}$$

For the Earth:

$$\begin{aligned} E_{\oplus} &= E_{\star} \times \frac{A_{\oplus}}{A_{surf}} \\ &= E_{\star} \times \frac{\pi \times R_{\oplus}^2}{4 \times \pi \times (4 \text{ ly})^2} \\ &= 8.938 \times 10^{49} \times \frac{\pi \times (6.378 \times 10^6 \text{ m})^2}{4 \times \pi \times (4 \text{ ly})^2} \\ &\simeq 6.5 \times 10^{29} \text{J} \end{aligned}$$

So the effective energy of the exploding star impacting the sun is $7.5 \times 10^{33} \text{J}$, and the effective energy impacting the earth is $6.5 \times 10^{29} \text{J}$.

3 What does this actually mean?

Now that we have these energies, what do they really mean? This is the hardest part to deal with. First lets do a pure energy to mass conversion.

At the sun:

$$\begin{aligned}m_{\odot eff} &= E_{\odot}/c^2 \\ &= 7.5 \times 10^{33} \text{J}/c^2 \\ &\simeq 8.3 \times 10^{16} \text{kg} \\ &\simeq 4.2 \times 10^{-14} M_{\odot} \\ &\simeq 1.4 \times 10^{-8} M_{\oplus}\end{aligned}$$

At the earth:

$$\begin{aligned}m_{\oplus eff} &= E_{\oplus}/c^2 \\ &= 6.5 \times 10^{29} \text{J}/c^2 \\ &\simeq 7.2 \times 10^{12} \text{kg} \\ &\simeq 3.6 \times 10^{-18} M_{\odot} \\ &\simeq 1.2 \times 10^{-12} M_{\oplus}\end{aligned}$$

These are fairly small mass values, but still we could use a clearer picture. Lets compare these values to the irradiance of the sun: $3.9 \times 10^{26} \text{W}$.

Energy hitting the sun:

$$\frac{E_{\odot}}{L_{\odot}} \simeq 1.9 \times 10^7 \text{ sec} \simeq 0.6 \text{ years}$$

Energy hitting the earth:

$$\frac{E_{\oplus}}{L_{\odot}} \simeq 1.7 \times 10^3 \text{ sec} \simeq 30 \text{ min}$$

Now, we can see a couple things here. First, the energy hitting the sun and earth is not that amazing. The amount of energy that would hit the sun is equivalent to the amount of energy the sun would emit in a little over a half a year. This is not an unreasonable amount of energy, but it is still enough energy to do damage. It could possibly wipe away the outer layers of the sun. Similarly the energy hitting the earth is equivalent to the amount of energy the sun will release in about 30 min. Without going into any great detail, we could imagine that if all the energy that the sun irradiates in 30 min was to hit the earth as a giant solar flare all at once, it would more than likely end all life on earth. I do however doubt that this would cause the planet to explode. I supposed it might be possible for the electromagnetic fields to wreak havoc with our iron core causing every volcano to go off at once and destroy the planet further, but I am not really a geophysicist, so I cannot speak to that end. But lets go even

further and assume that all of the energy hitting the sun or earth was converted to pure kinetic energy.

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2E}{m}}$$

Then the velocity given to the sun is given by

$$v = \sqrt{\frac{2E_{\odot}}{M_{\odot}}}$$

$$= \sqrt{\frac{2 \times 7.5 \times 10^{33}\text{J}}{2 \times 10^{30}\text{kg}}}$$

$$\simeq 86.8 \text{ m/s}$$

$$\simeq 194 \text{ mph}$$

and the velocity given to the earth is given by

$$v = \sqrt{\frac{2E_{\oplus}}{M_{\oplus}}}$$

$$= \sqrt{\frac{2 \times 6.5 \times 10^{29}\text{J}}{6 \times 10^{24}\text{kg}}}$$

$$\simeq 466 \text{ m/s}$$

$$\simeq 1000 \text{ mph}$$

Now considering the earth is already moving around the sun at approximately 30 km per second, these values are really not that impressive. Given our assumption that all of the energy that hits the earth would be perfectly converted to mechanical energy of the earth, this added energy would very slightly warp the orbit of the earth around the sun. This is trivial compared to the claim that such a supernova would destroy the galaxy! The impact on the sun is even more minimal. The sun is believed to be moving at approximately $2.2 \times 10^5 \text{ m/s}$ around the center of the galaxy. The added energy would have almost no affect whatsoever in comparison.

4 In Conclusion

While the writers chose a fascinating way to build the story, it is very unrealistic. Undoubtedly, a supernova of a $500M_{\odot}$ star near to our solar system would be devastating to us, but it would not have the affect implied by the movie. The

planet would more than likely not be completely destroyed, and the rest of the galaxy may not even realize that it ever even happened. All life on our little planet may not survive, but the earth would probably keep on orbiting in an otherwise normal, albeit lonely, path. In more realistic terms, the supernova of a more normal mass star at a more average distance would have little or no affect on most of the stars around it (again ignoring the effects of such phenomena as GRB). I hope this does not detract from anyone's movie going experience. The purpose of this document is more just to examine one astrophysical phenomenon in a fun movie. Please don't think this in any way detracted from my own movie going experience. In fact this may have even made it more entertaining since I had an excuse to play with astrophysics!